

# A Parallel System with Priority to Preventive Maintenance over Repair Subject to Maximum Operation and Repair Times

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## Abstract

In this work, a stress is given on the evaluation of performance measures of a parallel system under priority with necessary conditions on operation and repair times. The units are identical in nature subject to constant failure. There is a single server who visits the system immediately to rectify the faults whenever occurred in the system. The system undergoes for preventive maintenance after a pre-specific time 't' up to which no failure occurs. However, repair of the unit is done at its failure. And, the unit is replaced by new one in case its repair is not possible by the server in a given maximum repair time. The maintenance and repair activities are perfect. Priority is given to preventive maintenance of one unit over repair of the other. The random variables associated with failure, preventive maintenance, repair and replacement times are statistically independent. The failure time and the time by which unit undergoes for preventive maintenance and replacement follow negative exponential distribution, whereas the distributions for preventive maintenance, repair and replacement rates are taken as arbitrary with different probability density functions. The system is observed at suitable regenerative epochs by using semi-Markov process and regenerative point technique. Graphs are drawn to depict the behavior of some important reliability measures giving particular values to the parameters and costs. The profit of the system model has also been evaluated considering different cost policies.

**Keywords:** Parallel System, Priority, Preventive Maintenance, Maximum Operation and Repair Times and Performance Measures.

## 1. Introduction

The importance of parallel operation of components in repairable systems has been felt by the users in view of their load sharing capacity and the ability to provide better services for a reasonable period. And, so research work on stochastic modeling of such systems has been propagated by the scholars and engineers including Kumar et al. (2010) and Malik and Gitangali (2012). They assumed that system can work for a long period without requiring any type of maintenance. But, sometimes a system has to work in varying environmental conditions and so deteriorates due to continued operation and ageing. In such a situation, preventive maintenance of the system can be conducted after a maximum operation of time to slow the deteriorate process. Malik and Barak (2013) studied a cold stand by system with preventive maintenance and repair.

Further, the performance of a system can be enhanced by giving priority in repair disciplines and also replacing the failed component by new one in case their repair time is too long. Singh and Agrafoites (1995), Kumar and Malik (2012) and Malik (2013) analyzed systems with cold standby redundancy under the aspects of priority in repair disciplines, maximum operation and repair times.

While considering the practical situations in mind, here reliability measures of a parallel system have been evaluated using the concepts of priority, maximum operation and repair times. The units are identical in nature subject to constant failure. There is a single server who visits the

system immediately to rectify the faults which occur during operation of the system. The system undergoes for preventive maintenance after a pre-specific time 't' up to which no failure occurs. However, repair of the unit is done at its failure. And, the unit is replaced by new one in case its repair is not possible by the server in a given maximum repair time. The unit works as new after preventive maintenance and repair. Priority is given to preventive maintenance of one unit over repair of the other. The random variables associated with failure, preventive maintenance, repair and replacement times are statistically independent. The failure time and the time by which unit undergoes for preventive maintenance and replacement follow negative exponential distribution, whereas the distributions for preventive maintenance, repair and replacement rates are taken as arbitrary with different probability density functions. The system is observed at suitable regenerative epochs by using semi-Markov process and regenerative point technique to drive expressions for transition probabilities, mean sojourn times, mean time to system failure(MTSF), availability, busy period of the server due to repair, preventive maintenance and replacement, expected number of repairs, preventive maintenances and replacements and finally, the profit function. Graphs are drawn to depict the behavior of some important reliability measures giving particular values to the parameters and costs. The profit of the system model has also been evaluated considering different cost policies.

## 2. Notations:

$E$	: Set of regenerative states
$\bar{E}$	: Set of non-regenerative states
$\lambda$	: Constant failure rate
$\alpha_0$	: The rate by which system undergoes for preventive maintenance (called maximum constant rate of operation time)
$\beta_0$	: The rate by which system undergoes for replacement (called maximum constant rate of repair time)
$FUR/FWR$	: The unit is failed and under repair/waiting for repair
$FURp$	: The unit is failed and under replacement
$UPm$	: The unit is under preventive maintenance
$WPM$	: The unit is waiting for preventive maintenance
$FUR/FWR$	: The unit is failed and under repair / waiting for repair continuously from previous state
$FURP$	: The unit is failed and under replacement continuously from previous state
$UPM$	: The unit is under preventive maintenance continuously from previous state
$WPM$	: The unit is waiting for preventive maintenance continuously from previous state
$g(t)/G(t)$	: pdf/cdf of repair time of the unit
$f(t)/F(t)$	: pdf/cdf of preventive maintenance time of the unit
$r(t)/R(t)$	: pdf/cdf of replacement time of the unit
$q_{ij}(t)/Q_{ij}(t)$	: pdf / cdf of passage time from regenerative state $S_i$ to a regenerative state $S_j$ or to a failed state $S_j$ without visiting any other regenerative state in $(0, t]$
$q_{ij,kr}(t)/Q_{ij,kr}(t)$	: pdf/cdf of direct transition time from regenerative state $S_i$ to a regenerative state $S_j$ or to a failed state $S_j$ visiting state $S_k, S_r$ once in $(0, t]$
$M_i(t)$	: Probability that the system up initially in state $S_i \in E$ is up at time $t$ without visiting to any regenerative state
$W_i(t)$	: Probability that the server is busy in the state $S_i$ up to time 't' without making

any transition to any other regenerative state or returning to the same state via one or more non-regenerative states.

$\mu_i$  : The mean sojourn time in state  $S_i$  which is given by

$$\mu_i = E(T) = \int_0^{\infty} P(T > t) dt = \sum_j m_{ij},$$

where  $T$  denotes the time to system failure.

$m_{ij}$  : Contribution to mean sojourn time ( $\mu_i$ ) in state  $S_i$  when system transits directly

to state  $S_j$  so that  $\mu_i = \sum_j m_{ij}$  and  $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^*'(0)$

Ⓢ/© : Symbol for Laplace-Stieltjes convolution/Laplace convolution

\*/\*\* : Symbol for Laplace Transformation /Laplace Stieltjes Transformation

The possible transition states of the system model are shown in fig.1

### Transition State Diagram

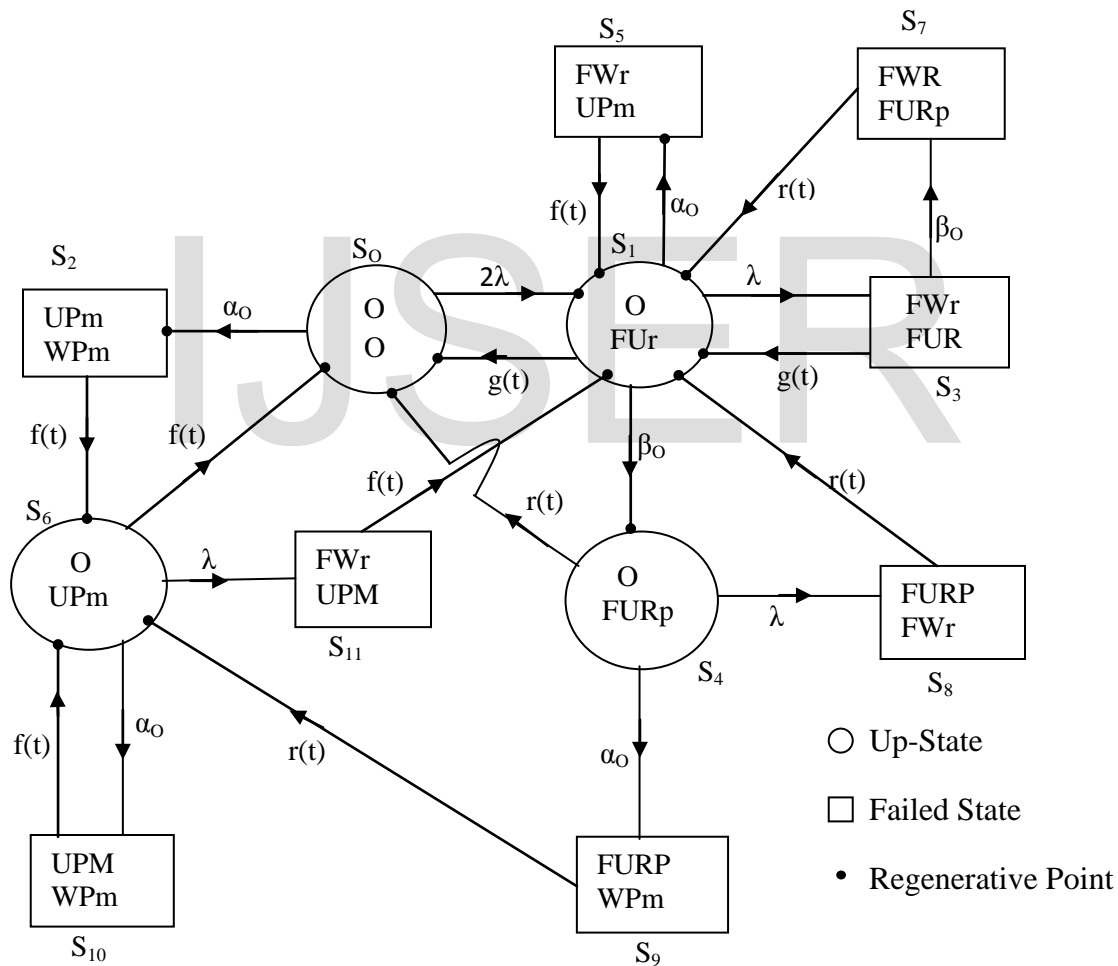


Fig. 1

### 3. Transition Probabilities and Mean Sojourn Times

Simple probabilistic considerations yield the following expressions for the non-zero elements

$$\begin{aligned}
 p_{ij} &= Q_{ij}(\infty) = \int_0^{\infty} q_{ij}(t)dt \quad \text{as} \quad (1) \\
 p_{01} &= \frac{2\lambda}{2\lambda + \alpha_0}, \quad p_{02} = \frac{\alpha_0}{2\lambda + \alpha_0}, \quad p_{10} = g^*(\lambda + \alpha_0 + \beta_0), \quad p_{13} = \frac{\lambda}{(\lambda + \alpha_0 + \beta_0)}(1 - g^*(\lambda + \alpha_0 + \beta_0)), \\
 p_{14} &= \frac{\beta_0}{(\lambda + \alpha_0 + \beta_0)}(1 - g^*(\lambda + \alpha_0 + \beta_0)), \quad p_{15} = \frac{\alpha_0}{(\lambda + \alpha_0 + \beta_0)}(1 - g^*(\lambda + \alpha_0 + \beta_0)), \quad p_{31} = g^*(\beta_0), \\
 p_{37} &= 1 - g^*(\beta_0), \quad p_{11.37} = \frac{\lambda}{(\lambda + \alpha_0 + \beta_0)}(1 - g^*(\beta_0))(1 - g^*(\lambda + \alpha_0 + \beta_0)), \quad p_{40} = r^*(\lambda + \alpha_0), \\
 p_{11.3} &= \frac{\lambda}{(\lambda + \alpha_0 + \beta_0)}g^*(\beta_0)(1 - g^*(\lambda + \alpha_0 + \beta_0)), \quad p_{41.8} = p_{48} = \frac{\lambda}{(\lambda + \alpha_0)}(1 - r^*(\lambda + \alpha_0)), \\
 p_{49} &= p_{46.9} = \frac{\alpha_0}{(\lambda + \alpha_0)}(1 - r^*(\lambda + \alpha_0)), \quad p_{6,10} = p_{66.10} = \frac{\alpha_0}{(\lambda + \alpha_0)}(1 - f^*(\lambda + \alpha_0)), \\
 p_{60} &= f^*(\lambda + \alpha_0), \quad p_{61.11} = p_{6,11} = \frac{\lambda}{(\lambda + \alpha_0)}(1 - f^*(\lambda + \alpha_0)), \\
 p_{26} &= p_{51} = p_{71} = p_{81} = p_{96} = p_{10,6} = p_{11,1} = 1 \quad (2)
 \end{aligned}$$

It can be easily verify that

$$p_{01} + p_{02} = p_{10} + p_{13} + p_{14} + p_{15} = p_{40} + p_{48} + p_{49} = p_{10} + p_{14} + p_{15} + p_{11.3} + p_{11.37} = p_{40} + p_{46.9} + p_{41.8} = p_{60} + p_{6,10} + p_{6,11} = p_{60} + p_{66.10} + p_{61.11} = 1$$

The mean sojourn times ( $\mu_i$ ) is in the state  $S_i$  are

$$\begin{aligned}
 \mu_0 &= m_{01} + m_{02}, \quad \mu_1 = m_{10} + m_{13} + m_{14} + m_{15}, \quad \mu_2 = m_{26}, \quad \mu_4 = m_{40} + m_{48} + m_{49}, \quad \mu_5 = m_{51}, \\
 \mu_6 &= m_{60} + m_{6,10} + m_{6,11}, \quad \mu_1 = m_{10} + m_{14} + m_{15} + m_{11.3} + m_{11.37}, \quad \mu_4 = m_{40} + m_{46.9} + m_{41.8}, \\
 \mu_6 &= m_{60} + m_{66.10} + m_{61.11} \quad (3)
 \end{aligned}$$

### 4. Reliability and Mean Time to System Failure (MTSF)

Let  $\phi_i(t)$  be the cdf of first passage time from regenerative state  $S_i$  to a failed state. Regarding the failed state as absorbing state, we have the following recursive relations for  $\phi_i(t)$ :

$$\begin{aligned}
 \Phi_0(t) &= Q_{01}(t) \oplus \Phi_1(t) + Q_{02}(t) \\
 \Phi_1(t) &= Q_{10}(t) \oplus \Phi_0(t) + Q_{14}(t) \oplus \Phi_4(t) + Q_{13}(t) + Q_{15}(t) \\
 \Phi_4(t) &= Q_{40}(t) \oplus \Phi_0(t) + Q_{49}(t) + Q_{48}(t) \quad (4)
 \end{aligned}$$

Taking LST of above relations (4) and solving for  $\Phi_0^{**}(s)$ , we have

$$R^*(s) = \frac{1 - \phi_0^{**}(s)}{s} \quad (5)$$

The reliability of the system model can be obtained by taking Inverse Laplace transform of (5). The mean time to system failure (MTSF) is given by

$$\text{MTSF} = \lim_{s \rightarrow 0} \frac{1 - \phi_0^{**}(s)}{s} = \frac{N}{D}, \quad (6)$$

where ,

$$N = \mu_0 + p_{01}\mu_1 + p_{01}p_{14}\mu_4 \text{ and } D = 1 - p_{01}p_{10} - p_{01}p_{14}p_{40} \quad (7)$$

### 5. Steady State Availability

Let  $A_i(t)$  be the probability that the system is in up-state at instant 't' given that the system entered regenerative state  $S_i$  at  $t = 0$ . The recursive relations for  $A_i(t)$  are given as:

$$\begin{aligned} A_0(t) &= M_0(t) + q_{01}(t) \odot A_1(t) + q_{02}(t) \odot A_2(t) \\ A_1(t) &= M_1(t) + q_{10}(t) \odot A_0(t) + q_{14}(t) \odot A_4(t) + q_{15}(t) \odot A_5(t) + (q_{11.3}(t) + q_{11.37}(t)) \odot A_1(t) \\ A_2(t) &= q_{26}(t) \odot A_6(t) \\ A_4(t) &= M_4(t) + q_{40}(t) \odot A_0(t) + q_{41.8}(t) \odot A_1(t) + q_{46.9}(t) \odot A_6(t) \\ A_5(t) &= q_{51}(t) \odot A_1(t) \\ A_6(t) &= M_6(t) + q_{60}(t) \odot A_0(t) + q_{66.10}(t) \odot A_6(t) + q_{61.11}(t) \odot A_1(t) \end{aligned} \quad (8)$$

Where

$$M_0(t) = e^{-(2\lambda + \alpha_0)t}, M_1(t) = e^{-(\lambda + \alpha_0 + \beta_0)t} \overline{G}(t), M_4(t) = e^{-(\lambda + \alpha_0)t} \overline{R}(t), M_6(t) = e^{-(\lambda + \alpha_0)t} \overline{F}(t) \quad (9)$$

Taking LT of above relations (8) and solving for  $A_0^*(s)$ . The steady state availability is given by

$$A_0(\infty) = \lim_{s \rightarrow 0} sA_0^*(s) = \frac{N_1}{D_1}, \quad (10)$$

where

$$N_1 = (\mu_0 X + (\mu_1 + \mu_4 p_{14}) Y + \mu_6 Z) \quad (11)$$

$$D_1 = X(\mu_0 + \mu_2 p_{02}) + Y(\mu_1 + \mu_4 p_{14} + \mu_5 p_{15}) + Z\mu_6 \quad (12)$$

### 6. Busy Period Analysis for Server

#### (a) Due to Repair

Let  $B_i^R(t)$  be the probability that the server is busy in repair the unit at an instant 't' given that the system entered regenerative state  $S_i$  at  $t=0$ . The recursive relations for  $B_i^R(t)$  are as follows:

$$\begin{aligned} B_0^R(t) &= q_{01}(t) \odot B_1^R(t) + q_{02}(t) \odot B_2^R(t) \\ B_1^R(t) &= W_1(t) + q_{10}(t) \odot B_0^R(t) + (q_{11.37}(t) + q_{11.3}(t)) \odot B_1^R(t) + q_{14}(t) \odot B_4^R(t) + q_{15}(t) \odot B_5^R(t) \\ B_2^R(t) &= q_{26}(t) \odot B_6^R(t) \\ B_4^R(t) &= q_{40}(t) \odot B_0^R(t) + q_{41.8}(t) \odot B_1^R(t) + q_{46.9}(t) \odot B_6^R(t) \\ B_5^R(t) &= q_{51}(t) \odot B_1^R(t) \\ B_6^R(t) &= q_{60}(t) \odot B_0^R(t) + q_{61.11}(t) \odot B_1^R(t) + q_{66.10}(t) \odot B_6^R(t) \end{aligned} \quad (13)$$

where,

$$W_1(t) = e^{-(\lambda + \alpha_0 + \beta_0)t} \overline{G}(t) + (\lambda e^{-(\lambda + \alpha_0 + \beta_0)t} \odot 1) \overline{G}(t) \quad (14)$$

Taking LT of above relations (13) and solving for  $B_0^{R*}(s)$ . The time for which server is busy due to repair is given by

$$B_0^R(\infty) = \lim_{s \rightarrow 0} s B_0^{R*}(s) = \frac{N_2}{D_1}, \quad (15)$$

Where

$$N_2 = W_1^*(0)Y \quad \text{and} \quad D_1 \text{ is already mentioned.} \quad (16)$$

### (b) Due to Replacement

Let  $B_i^{Rp}(t)$  be the probability that the server is busy in replacement the unit at an instant 't' given that the system entered regenerative state  $S_i$  at  $t=0$ . The recursive relations for  $B_i^{Rp}(t)$  are as follows:

$$\begin{aligned} B_0^{Rp}(t) &= q_{01}(t) \odot B_1^{Rp}(t) + q_{02}(t) \odot B_2^{Rp}(t) \\ B_1^{Rp}(t) &= q_{10}(t) \odot B_0^{Rp}(t) + (q_{11.37}(t) + q_{11.3}(t)) \odot B_1^{Rp}(t) + q_{14}(t) \odot B_4^{Rp}(t) + q_{15}(t) \odot B_5^{Rp}(t) \\ B_2^{Rp}(t) &= q_{26}(t) \odot B_6^{Rp}(t) \\ B_4^{Rp}(t) &= W_4(t) + q_{40}(t) \odot B_0^{Rp}(t) + q_{41.8}(t) \odot B_1^{Rp}(t) + q_{46.9}(t) \odot B_6^{Rp}(t) \\ B_5^{Rp}(t) &= q_{51}(t) \odot B_1^{Rp}(t) \\ B_6^{Rp}(t) &= q_{60}(t) \odot B_0^{Rp}(t) + q_{61.11}(t) \odot B_1^{Rp}(t) + q_{66.10}(t) \odot B_6^{Rp}(t) \end{aligned} \quad (17)$$

where,

$$W_4(t) = e^{-(\lambda+\alpha_0)t} \overline{R}(t) + (\lambda e^{-(\lambda+\alpha_0)t} \odot 1) \overline{R}(t) \quad (18)$$

Taking LT of above relations (17) and solving for  $B_0^{Rp*}(s)$ . The time for which server is busy due to replacement is given by

$$B_0^{Rp}(\infty) = \lim_{s \rightarrow 0} s B_0^{Rp*}(s) = \frac{N_3}{D_1} \quad (19)$$

Where

$$N_3 = W_4^*(0) p_{14} Y \quad \text{and} \quad D_1 \text{ is already mentioned.} \quad (20)$$

### (c) Due to Preventive Maintenance

Let  $B_i^P(t)$  be the probability that the server is busy in preventive maintenance the unit at an instant 't' given that the system entered regenerative state  $S_i$  at  $t=0$ . The recursive relations for  $B_i^P(t)$  are as follows:

$$\begin{aligned} B_0^P(t) &= q_{01}(t) \odot B_1^P(t) + q_{02}(t) \odot B_2^P(t) \\ B_1^P(t) &= q_{10}(t) \odot B_0^P(t) + (q_{11.37}(t) + q_{11.3}(t)) \odot B_1^P(t) + q_{14}(t) \odot B_4^P(t) + q_{15}(t) \odot B_5^P(t) \\ B_2^P(t) &= W_2(t) + q_{26}(t) \odot B_6^P(t) \\ B_4^P(t) &= q_{40}(t) \odot B_0^P(t) + q_{41.8}(t) \odot B_1^P(t) + q_{46.9}(t) \odot B_6^P(t) \\ B_5^P(t) &= W_5(t) + q_{51}(t) \odot B_1^P(t) \\ B_6^P(t) &= W_6(t) + q_{60}(t) \odot B_0^P(t) + q_{61.11}(t) \odot B_1^P(t) + q_{66.10}(t) \odot B_6^P(t) \end{aligned} \quad (21)$$

Where,

$$W_2(t) = W_5(t) = \overline{F(t)}, \quad W_6(t) = e^{-(\lambda+\alpha_0)t} \overline{F(t)} + (\alpha_0 e^{-(\lambda+\alpha_0)t} \odot 1) \overline{F(t)} + (\lambda e^{-(\lambda+\alpha_0)t} \odot 1) \overline{F(t)} \quad (22)$$

Taking LT of above relations (21) and solving for  $B_0^{P*}(s)$ . The time for which server is busy due to preventive maintenance is given by

$$B_0^P(\infty) = \lim_{s \rightarrow 0} s B_0^{P*}(s) = \frac{N_4}{D_1}, \quad (23)$$

Where,

$$N_4 = W_2^*(0) p_{02} X + W_5^*(0) p_{15} Y + W_6^*(0) Z \text{ and } D_1 \text{ is already mentioned.} \quad (24)$$

### 7. Expected Number of Repairs

Let  $R_i(t)$  be the expected number of repairs by the server in  $(0, t]$  given that the system entered the regenerative state  $S_i$  at  $t = 0$ . The recursive relations for  $R_i(t)$  are given as:

$$\begin{aligned} R_0(t) &= Q_{01}(t) \odot R_1(t) + Q_{02}(t) \odot R_2(t) \\ R_1(t) &= Q_{10}(t) \odot (1 + R_0(t)) + Q_{11.3}(t) \odot (1 + R_1(t)) + Q_{11.37}(t) \odot R_1(t) + Q_{14}(t) \odot R_4(t) + Q_{15}(t) \odot R_5(t) \\ R_2(t) &= Q_{26}(t) \odot R_6(t) \\ R_4(t) &= Q_{40}(t) \odot R_0(t) + Q_{41.8}(t) \odot R_1(t) + Q_{46.9}(t) \odot R_6(t) \\ R_5(t) &= Q_{51}(t) \odot R_1(t) \\ R_6(t) &= Q_{60}(t) \odot R_0(t) + Q_{61.11}(t) \odot R_1(t) + Q_{66.10}(t) \odot R_6(t) \end{aligned} \quad (25)$$

Taking LST of above relations (25) and solving for  $R_0^{**}(s)$ . The expected no. of repairs per unit time by the server are giving by

$$R_0(\infty) = \lim_{s \rightarrow 0} s R_0^{**}(s) = \frac{N_5}{D_1}, \quad (26)$$

where

$$N_5 = (p_{10} + p_{11.3}) Y \text{ and } D_1 \text{ is already mentioned.} \quad (27)$$

### 8. Expected Number of Replacements

Let  $Rp_i(t)$  be the expected number of replacements by the server in  $(0, t]$  given that the system entered the regenerative state  $S_i$  at  $t = 0$ . The recursive relations for  $Rp_i(t)$  are given as:

$$\begin{aligned} Rp_0(t) &= Q_{01}(t) \odot Rp_1(t) + Q_{02}(t) \odot Rp_2(t) \\ Rp_1(t) &= Q_{10}(t) \odot Rp_0(t) + Q_{11.37}(t) \odot (1 + Rp_1(t)) + Q_{11.3}(t) \odot Rp_1(t) + Q_{14}(t) \odot Rp_4(t) + Q_{15}(t) \odot Rp_5(t) \\ Rp_2(t) &= Q_{26}(t) \odot Rp_6(t) \\ Rp_4(t) &= Q_{40}(t) \odot (1 + Rp_0(t)) + Q_{41.8}(t) \odot (1 + Rp_1(t)) + Q_{46.9}(t) \odot (1 + Rp_6(t)) \\ Rp_5(t) &= Q_{51}(t) \odot Rp_1(t) \\ Rp_6(t) &= Q_{60}(t) \odot Rp_0(t) + Q_{61.11}(t) \odot Rp_1(t) + Q_{66.10}(t) \odot Rp_6(t) \end{aligned} \quad (28)$$

Taking LST of above relations (28) and solving for  $Rp_0^{**}(s)$ . The expected no. of replacements per unit time by the server are giving by

$$Rp_0(\infty) = \lim_{s \rightarrow 0} sRp_0^{**}(s) = \frac{N_6}{D_1}, \tag{29}$$

where

$$N_6 = (p_{14} + p_{11.37})Y \quad \text{and} \quad D_1 \text{ is already mentioned.} \tag{30}$$

### 9. Expected Number of Preventive Maintenances

Let  $P_i(t)$  be the expected number of preventive maintenance by the server in  $(0, t]$  given that the system entered the regenerative state  $S_i$  at  $t = 0$ . The recursive relations for  $P_i(t)$  are given as:

$$\begin{aligned} P_0(t) &= Q_{01}(t) \otimes P_1(t) + Q_{02}(t) \otimes P_2(t) \\ P_1(t) &= Q_{10}(t) \otimes P_0(t) + (Q_{11.3}(t) + Q_{11.37}(t)) \otimes P_1(t) + Q_{14}(t) \otimes P_4(t) + Q_{15}(t) \otimes P_5(t) \\ P_2(t) &= Q_{26}(t) \otimes (1 + P_6(t)) \\ P_4(t) &= Q_{40}(t) \otimes P_0(t) + Q_{41.8}(t) \otimes P_1(t) + Q_{46.9}(t) \otimes P_6(t) \\ P_5(t) &= Q_{51}(t) \otimes (1 + P_1(t)) \\ P_6(t) &= Q_{60}(t) \otimes (1 + P_0(t)) + Q_{61.11}(t) \otimes (1 + P_1(t)) + Q_{66.10}(t) \otimes (1 + P_6(t)) \end{aligned} \tag{31}$$

Taking LST of above relations (31) and solving for  $P_0^{**}(s)$ . The expected no. of preventive maintenances per unit time by the server are giving by

$$P_0(\infty) = \lim_{s \rightarrow 0} sP_0^{**}(s) = \frac{N_7}{D_1}, \tag{32}$$

Where

$$N_7 = p_{02}X + p_{15}Y + Z \quad \text{and} \quad D_1 \text{ is already mentioned.} \tag{33}$$

And

$$\begin{aligned} X &= (1 - p_{66.10})(p_{10} + p_{14}p_{40}) + p_{14}p_{49}p_{60} \\ Y &= \frac{\lambda(2\lambda + 2\alpha + \alpha_0)}{(2\lambda + \alpha_0)(\alpha + \lambda + \alpha_0)} \\ Z &= \frac{\beta_0\alpha_0(2\lambda + \alpha_0)(2\lambda + \alpha_0 + \beta) + 2\lambda\alpha_0(\theta + \lambda + \alpha_0 + \beta_0)(\beta + \lambda + \alpha_0)}{(\theta + \lambda + \alpha_0 + \beta_0)(\beta + \lambda + \alpha_0)(2\lambda + \alpha_0)^2} \end{aligned}$$

### 10. Profit Analysis

The profit incurred to the system model in steady state can be obtained as

$$P_1 = K_0A_0 - K_1B_0^R - K_2B_0^{Rp} - K_3B_0^P$$

$$P_2 = K_0A_0 - K_4R_0 - K_5Rp_0 - K_6P_0$$

Where

$P_1$  = Profit of the system model after reducing cost of the repair activities of server

$P_2$  = Profit of the system model after reducing cost of expected number of repair activities

$K_0$  = Revenue per unit up-time of the system

$K_1$  = Cost per unit time for which server is busy due to repair

$K_2$  = Cost per unit time for which server is busy due to replacement

$K_3$  = Cost per unit time for which server is busy due to preventive maintenance



$K_4$  = Cost per unit time repair  
 $K_5$  = Cost per unit time replacement  
 $K_6$  = Cost per unit time preventive maintenance

## 11. Conclusion

The reliability measures of a parallel system giving priority to preventive maintenance over repair have been obtained for the particular case  $g(t) = \theta e^{-\theta t}$ ,  $r(t) = \beta e^{-\beta t}$  and  $f(t) = \alpha e^{-\alpha t}$ . To make the study more concrete and effective, the graphs for MTSF, availability and profit are drawn with respect to failure rate ( $\lambda$ ) for fixed values of other parameters as shown respectively in figures 2, 3, 4 and 5. The results indicate that MTSF, availability and profit go on decreasing with the increase of failure rate ( $\lambda$ ) and the rate ( $\alpha_0$ ) by which the unit undergoes for preventive maintenance. However, their values increase with the increase of repair rate ( $\theta$ ) and replacement rate ( $\beta$ ). Further, MTSF and availability increase as and when the rate ( $\beta_0$ ) increases while profit declines. The profit of the system model has also been obtained under two aspects of costs-one is by considering cost of busy period of the server due to repair activities and the other by giving separate cost of each repair activity to the server. It is analyzed that the system would be more profitable if cost is paid to the server for his busy period rather than cost to each repair activity.

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